

A Self Study Guide for **Strength of Materials**

(For Under-Graduate Engineering Students)

by

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PREFACE

I must thank CMD of MADE EASY Group, **Mr. B. Singh** for providing me an opportunity to reach out to the Student Community at large through my present book “**A Self Study Guide for Strength of Materials**”. Students may be benefitted from my 60 years of teaching /research experience through this book.

Questions in the book are designed on the pattern of questions that are being asked in university examinations and competitive examinations of UPSC/GATE/PSUs.

The book has been thoroughly reviewed and questions from competitive examinations for the last 2 years have been added, in this book.

Further improvements in the text book will be made after getting the response from the students.

Any error in printing or calculations pointed out by the reader will be acknowledged with thanks by the author.

Dr. U. C. Jindal

Author

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Simple Stresses and Strains

CHAPTER

1

In this chapter we will learn about (i) different types of stresses as direct stress, shear stress, volumetric stress; (ii) different types of strains as longitudinal strain, shear strain; (iii) different types of elastic modulus as Young's modulus, shear modulus; bulk modulus; (iv) different types of loads as gradual load, sudden load, impact load; (v) strain energy absorbed in a body while loading; (vi) safe load and factor of safety for a body so that it does not break during loading; (vii) different types of bars as a bar of uniform section, tapered bar, stepped bar depending upon their applications in industry; (viii) different types of mechanical properties as strength, stiffness, ductility, toughness etc.

A student should remember that concepts explained in this chapter will be used in all the remaining chapters of the book. So, he has to learn by heart about different types of stresses, strains and safe limit of stresses.

1.1 Direct Load and Direct Stresses or Normal Loads and Normal Stresses

Any load or force which is **applied perpendicular to the section of a bar** is known as direct load or normal load.

Figure 1.1(a) shows a circular bar of **diameter, d and length, L** . Bar is supported on a flat ground and at the top, there is a flat plate. A weight W is placed on the flat plate. The purpose of the flat plate is to uniformly distribute the load W , on the bar. There is equal and opposite reaction, $R = W$, to maintain equilibrium. In Engineering Mechanics, a student learns about equilibrium. **This type of load (pointing towards the section of the bar) is known as compressive load.**

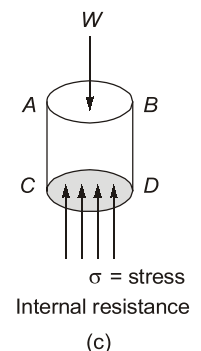
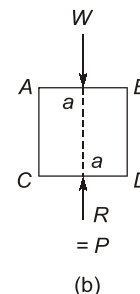
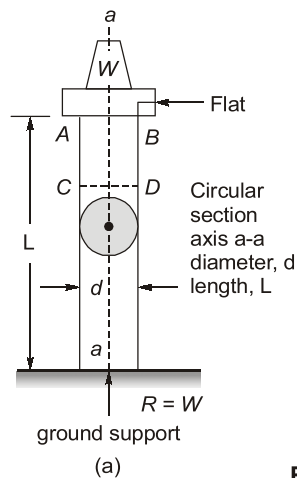


Figure : 1.1

The compressive load decreases the length of the bar and increases its diameter. Total effect is reduction in volume of the bar. Reduction in volume increases the density of the material of the bar and improves its strength slightly.

In figure 1.1 (b), a portion $ABCD$ of the bar is considered. On the upper section, there is load W , on the internal section CD of the bar, there is internal reaction $R = W$. **The internal resistance of the bar per unit area is termed as stress, σ .**

Diameter of the bar = d

$$\text{Area of cross section} = \frac{\pi}{4} d^2 = A$$

$$\text{Internal resistance, } R = W$$

$$\text{Stress} = \frac{R}{A} = \frac{W}{A} = \frac{4W}{\pi d^2} \quad \dots(i)$$

Note that internal resistance, $R = W$ is pointing towards the section, and the stress, σ is pointing towards the section. This type of stress is known as compressive stress. By convention, compressive stress is taken as **negative direct stress**.

$$\text{Stress, } \sigma = -\frac{4W}{\pi d^2} \text{ (compressive)}$$

It is very simple and easy to apply a compressive load. But to **apply a tensile load**, which stretches the bar, requires some **attachment at both the ends of the bar**.

Figure 1.2 (a) shows a circular bar. Two collars of diameter D are provided at the ends, so that the force (load) can be applied on the shoulders, through a machine (clamps or grips) with sharp wedges grip the collars at both the ends. While the lower end remains stationary, other end is moved upward with the help of hydraulic pressure provided by the testing machine. Bar has uniform section of diameter, d over the length, L called as **gauge length**. When the hydraulic force is applied on upper load, the bar develops internal resistance equal to the applied hydraulic force, P . An extensometer is clamped on the bar on the gauge length. A record of continuously increasing force P and change in length δL is made and graph is plotted between P and δL to study various mechanical properties of the material of the bar. In this case, force P and internal resistance P are pointing away from the sections on which they are applied.

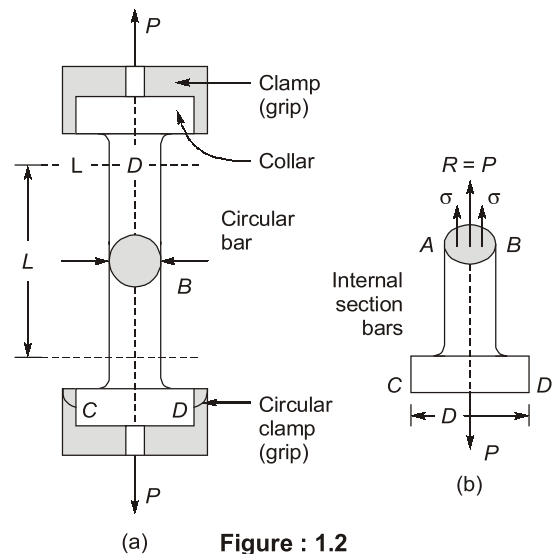


Figure : 1.2

This type of force or load is termed as **tensile direct load**. Internal resistance P per unit area is known as tensile stress, σ . By convention this stress is taken as **positive normal stress**. This tensile stress increases the length, while the diameter of the bar is slightly reduced. Volume of the bar is slightly increased and the density of the material is slightly reduced, thus decreasing the strength of the materials, which depends on the density of the material.

$$\text{Direct load or force} = P; \quad \text{Diameter of the bar} = d$$

$$\text{Area of cross-section, } A = \frac{\pi}{4} d^2$$

$$\text{Tensile stress, } \sigma = +\frac{4P}{\pi d^2} = \frac{P}{A}$$

The load on a bar or any body is applied as a distributed load. The resultant of the distributed load passes through the axis of the bar.

But, in general, numerical problems of the subject of strength of material, end supports or clamps are not shown and simple line diagrams are made as shown in figure 1.3.

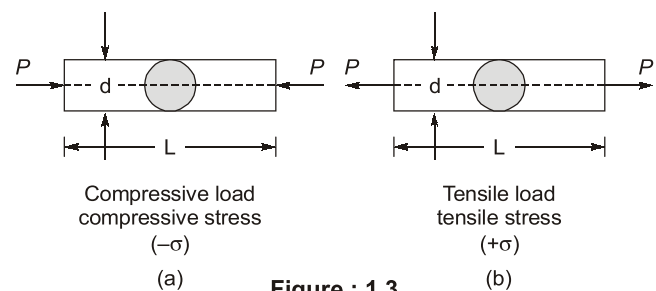


Figure : 1.3

REMEMBER

- I. A compressive direct load acts perpendicular to the plane and points towards the planes producing compressive stress ($-\sigma$)
- II. A tensile direct load acts perpendicular to the plane and points away from the plane producing tensile, stress ($+\sigma$).

1.2 Longitudinal Strain and Lateral Strain

Consider a bar of length, L and diameter, d subjected to a compressive force P as shown in Figure 1.4. Original length, L is reduced to final length, L' (where $L' < L$) and original diameter, d is increased to d' (where $d' > d$).

Change in length, $\delta L = L' - L$ (negative)

Change in diameter, $\delta d = d' - d$ (positive)

Longitudinal strain (axial), $\epsilon_a = \frac{L' - L}{L} = \left[\frac{\delta L}{L} \right]$

Lateral strain, $\epsilon_l = \frac{\delta d}{d} = \frac{d' - d}{d}$

$$\text{Ratio} = \frac{\epsilon_l}{\epsilon_a} = \left(\frac{d' - d}{d} \right) \times \left(\frac{L}{L' - L} \right) = \frac{\delta d}{d} \times \frac{L}{\delta L}$$

$$= -\nu \text{ (Poisson's ratio)}$$

Ratio of lateral strain/axial strain is known as **Poisson's ratio**. Poisson was a scientist who gave this concept of ratio of lateral strain/longitudinal strain. Poisson's ratio depends on the material of this bar.

Similarly, consider a bar of length, L and diameter, d subjected to an axial tensile force, P as shown in Figure 1.5. Under the tensile load, length of bar is increased to L' ($L' > L$) and diameter of the bar is reduced to d' ($d' < d$).

Change in axial length, $\delta L = L' - L$ (+ve)

Longitudinal strain (axial stress), $\epsilon_a = \frac{L' - L}{L} = + \frac{\delta L}{L}$
= a positive quantity

Lateral strain, $\epsilon_l = \frac{d' - d}{d} = - \frac{\delta d}{d}$ = a negative quantity

$$\text{Ratio} = \frac{\epsilon_l}{\epsilon_a} = \frac{\delta d}{d} \times \frac{L}{\delta L} = -\nu \text{ (Poisson's ratio)}$$

Ratio of lateral strain/longitudinal strain is termed as **Poisson's ratio**.

For some common engineering materials, Poisson's ratio is given in Table 1.1.

Table : Poisson's Ratio

Material	ν	Material	ν
Aluminium	0.33	Nickel	0.29
Brass	0.35	Steel	0.30
Cast Iron	0.25	Titanium	0.34
Copper	0.35	Tungsten	0.25

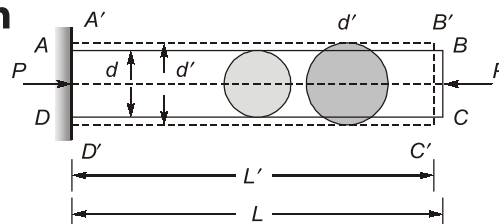


Figure 1.4 : Bar subjected to compressive load

(negative strain)

(positive strain)

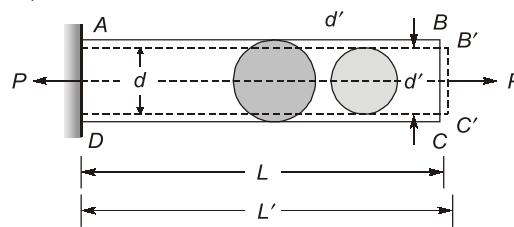


Figure 1.5 : Bar subjected to axial tensile force

Remember that ratio of lateral strain/longitudinal strain is always negative. If longitudinal strain is positive, the lateral strain will be negative and vice-versa.

1.3 Hooke's Law

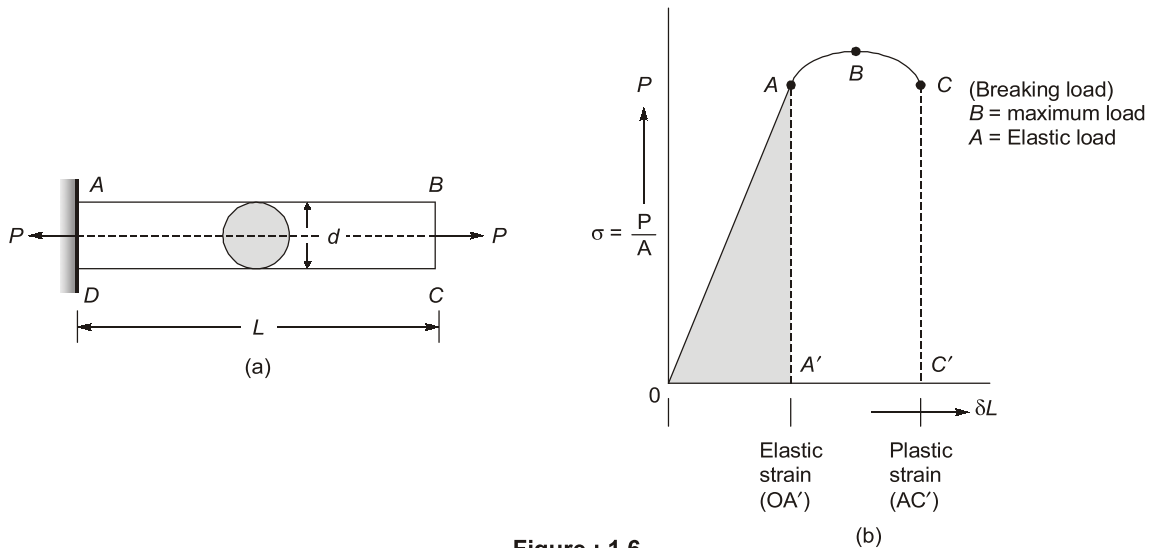


Figure : 1.6

A bar of some material, axial length, L and diameter, d is subjected to a tensile load, P through a testing machine. The load is continuously increasing till the bar breaks. Simultaneously the increase in length is noted. A graph is plotted between P and change in length, δL as shown in figure 1.6 (b). From the graph, OA is linear, load is increasing and change in length is also linearly increasing. The graph becomes non-linear, as shown by curve ABC , point B shows maximum load on the bar and point C shows breaking load on the bar. OA is linear, **A is elastic limit of material.**

As per Hooke's law, stress is proportional to strain within the elastic limit.

From graph upto elastic limit,

$$P \propto \delta L$$

$$\text{Load} \propto \text{Change in length}$$

$$\therefore \frac{P}{A} \propto \frac{\delta L}{L}$$

where A is original area of cross-section $A = \frac{\pi}{4}d^2$ and L is original length. Both are constants, i.e.,

$$\sigma, \text{ stress} \propto \epsilon, \text{ strain}$$

$$\text{or, } \sigma = E\epsilon$$

where E is known as proportionality constant. E is also known. Young's modulus of elasticity, first given by Prof. Young.

E = Elastic modulus

$$= \frac{\sigma}{\epsilon} = \frac{\text{stress}}{\text{strain}} = \frac{\text{direct stress}}{\text{longitudinal strain}}$$

$$\text{or, strain, } \epsilon = \frac{\sigma}{E}$$

Note that strain is a material property, while stress, σ depends only on the applied load. For various materials, for the same stress, σ , strain ϵ will be different depending on the modulus of elasticity of the material.

Within the elastic limit, strain ϵ is recoverable. But beyond elastic limit, in the plastic stage, on removing the load from the bar there will be **permanent set in the material** (i.e., residual strain).

Table shows the values of elastic modulus of different engineering materials.

Material	E in kN/mm ²	Material	E in kN/mm ²
Aluminium	70	Nickel	45
Brass	100	Steel	210
Cast Iron	105	Titanium	107
Copper	110	Tungsten	408

Question 1.1

A steel flat of rectangular section 30 mm × 20 mm, 600 mm long is subjected to an axial tensile force of 30 kN. Determine the changes in length and breadth and thickness of the flat. Given $E = 210 \text{ kN/mm}^2$, Poisson's ratio, $\nu = 0.3$ (for steel)

Solution: Figure 1.7 shows steel flat of length, $L = 600 \text{ mm}$; breadth, $b = 30 \text{ mm}$; thickness, $t = 20 \text{ mm}$.

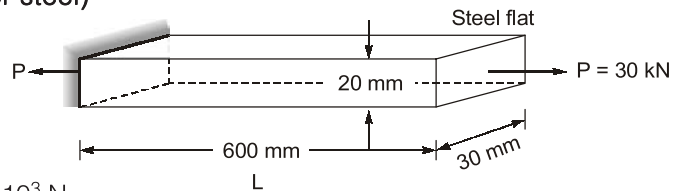


Figure : 1.7

Axial load, $P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$
 Area of cross-section, $A = b \times t = 30 \times 20 = 600 \text{ mm}^2$

Tensile stress, $\sigma = \frac{P}{A} = \frac{30 \times 10^3}{600}$
 $= 50 \text{ N/mm}^2$ or 50 mPa
 $= 50 \text{ Mega Pascal}$

where $1 \text{ Pascal} = \frac{1 \text{ N}}{\text{m}^2}$
 $= \frac{1.0 \text{ N}}{10^6} \text{ N/mm}^2$

Mega Pascal $= \frac{1 \times 10^6}{10^6}$
 $= 1 \text{ N/mm}^2$

Young's modulus, $E = 210 \text{ kN/mm}^2$

Longitudinal strain, $\epsilon_a = \frac{\sigma}{E} = \frac{50}{210 \times 10^3}$
 $= 0.238 \times 10^{-3}$ (positive strain)

Change in length, $\delta L = \epsilon_a \times L$
 $= 0.238 \times 10^{-3} \times 600$
 $= 0.0143 \text{ mm}$

Lateral strain, $\epsilon_l = -\nu \times \epsilon_a = -0.3 \times 0.238 \times 10^{-3}$
 $= -7.14 \times 10^{-5}$

Change in breadth, $\delta b = \epsilon_l \times b$

$$\begin{aligned}
 &= -7.14 \times 10^{-5} \times 30 \\
 &= 2.142 \times 10^{-3} \text{ mm} \\
 \text{Change in thickness, } \delta t &= t \times \epsilon_l \\
 &= -7.14 \times 10^{-5} \times 20 \\
 &= 1.428 \times 10^{-3} \text{ mm}
 \end{aligned}$$

Question 1.2 A round cast iron bar of diameter 30 mm is subjected to axial compressive load of 40 kN. Length of bar is 500 mm. If $E = 105 \text{ kN/mm}^2$ and $\nu = 0.25$ for cast iron, what are the changes in its length and diameter.

Solution: Figure 1.8 shows a cast iron bar of length, $L = 500 \text{ mm}$, diameter, $d = 30 \text{ mm}$ subjected to axial compressive load, $P = 40 \text{ kN}$.

$$\begin{aligned}
 \text{Diameter, } d &= 30 \text{ mm} \\
 \text{Area of cross-section, } A &= \frac{\pi}{4} \times d^2 \\
 &= \frac{\pi}{4} \times 30^2 = 706.86 \text{ mm}^2 \\
 \text{Compressive load, } P &= 40 \text{ kN} \\
 \text{Compressive stress, } \sigma &= \frac{P}{A} = \frac{40 \times 10^3}{706.86} \\
 &= 56.59 \text{ N/mm}^2 \\
 \text{Young's modulus, } E &= 105 \text{ kN/mm}^2 \\
 \text{Axial strain, } \epsilon_a &= \frac{\sigma}{E} = \frac{56.59}{105 \times 10^3} \\
 &= 0.539 \times 10^{-3} \\
 \text{Lateral strain, } \epsilon_l &= -\nu \epsilon_a \\
 &= -0.25 \times 0.539 \times 10^{-3} \\
 &= -0.135 \times 10^{-3} \\
 \text{Change in length, } \delta L &= \epsilon_a \times L = 0.539 \times 10^{-3} \times 500 \\
 &= 0.269 \text{ mm} \\
 \text{Change in diameter, } \delta d &= \epsilon_l \times d \\
 &= -0.135 \times 10^{-3} \times 30 = -4.05 \times 10^{-3} \text{ mm}
 \end{aligned}$$

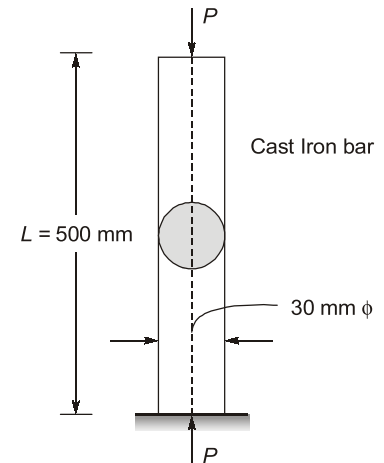


Figure : 1.8

Practice Q.1.1 A square bar $20 \times 20 \text{ mm}$ of copper, 200 mm long is subjected to axial force, $P = +24 \text{ kN}$. If $E = 102 \text{ kN/mm}^2$ and Poisson's ratio, $\nu = 0.3$, determine change in length and side of the bar.

Ans. $[+0.117 \text{ mm}, -4.116 \times 10^{-3} \text{ mm}]$

Practice Q.1.2 A round steel bar of diameter 25 mm , length $L = 250 \text{ mm}$ is subjected to axial compressive load of 45 kN . If $E = 208 \text{ kN/mm}^2$ and $\nu = 0.3$ for steel, determine changes in length and diameter of the bar.

Ans. $[-0.3305 \text{ mm}, +0.0099 \text{ mm}]$

1.4 Shear Stress and Shear Strain

Shear force or shear stress acts tangential to the plane of loading of a body. Shear strain produced by shear stress changes the shape of the body.

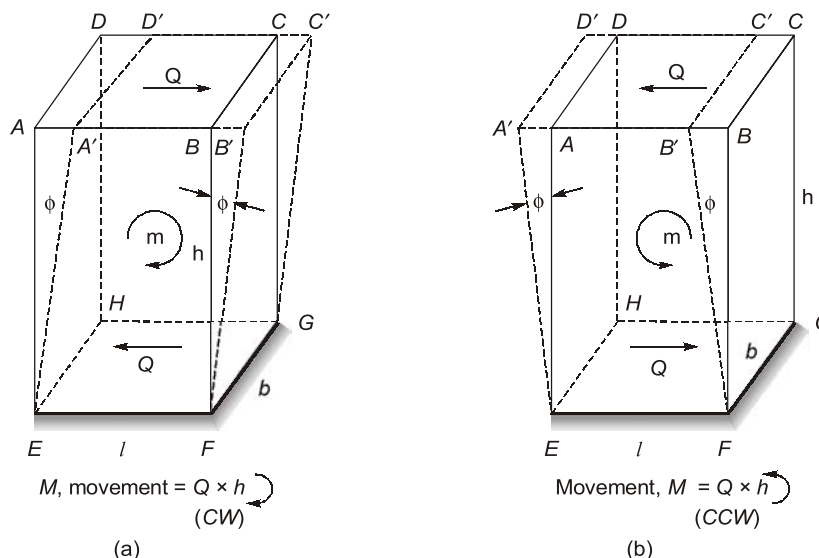


Figure : 1.9

Figure 1.9 (a) shows a rectangular block $ABCDEFGH$, with dimensions $l \times b \times h$ (as shown). Bottom face $EFGH$ is fixed, while a shear force \vec{Q} applied on the top face $ABCD$. On the bottom surface, there will be equal and opposite reaction. Note that $Q \times h$ is moment acting on the body, producing shear strain ϕ ($\angle BFB'$) shape of the face ΔBFE is changed from rectangular to parallelogram.

This type of shear force which tends to rotate the body in clockwise (CW) direction is taken as **positive shear force and shear stress** = shear force/area = $Q/l \times b = \tau$, a positive shear stress.

Shear strain ϕ = shear angle ϕ is proportional to shear stress, τ , i.e., shear stress, $\tau \propto \phi$, clear strain, as per Hooke's law (within the elastic limit shear stress is linearly proportional to shear strain).

Note further that shear angle ϕ is very small, much less than 1° (within the elastic limit). Note that angle ϕ is in radian.

$$\text{Shear strain, } \phi = \tan \frac{BB'}{BH} = \tan \frac{BB'}{h} \simeq \phi$$

because ϕ is very much small.

$$\tau \propto \phi$$

or

$$\tau = G\phi$$

G is proportionality constant or shear modulus in this case.

or Shear modulus,

$$G = \frac{\tau}{\phi} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

Similarly, take figure (b), shear force Q acts on top face, \vec{Q} and on bottom face \vec{Q} is equal and opposite reaction, \vec{Q} . This shear force tends to rotate the body in anti-clockwise direction.

Shear stress, $\tau = \frac{Q}{l \times b}$ is **negative shear force**, tends to rotate the body in anti-clockwise direction.

Shear stress, $\tau \propto \phi$, shear strain (shear angle)
 $\tau = G\phi$, where G is shear modulus of the material of the body

or $G = \frac{\tau}{\phi} = \frac{\text{Shear stress}}{\text{Shear strain}}$

REMEMBER

1. Shear stress which tends to rotate the body in **clockwise direction** in a position shear stress.
2. Shear stress which tends to rotate the body in **anticlockwise direction** is a **negative shear stress**.
3. $\tau \propto \phi$, shear stress is linearly proportional to shear strain, within the elastic limit.
4. Shear modulus, G = shear stress, τ /shear strain, ϕ .

Question 1.3

A rectangular block of aluminium of size 60 mm × 40 mm × 100 mm is subjected to a shear force of 48 kN on face 60 × 40 mm. If G of aluminium = 25 kN/mm², what are shear stress, shear strain on block?

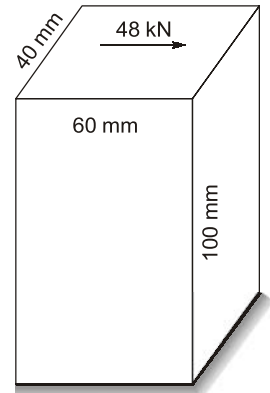
Solution: Figure 1.10 shows block 60 × 40 × 100 mm, on face 60 × 40 mm shear force 48 kN acts as shown.

Shear force, $Q = 48 \text{ kN}$
 Area of top face, $A = 60 \times 40 = 2400 \text{ mm}^2$

Shear stress, $\tau = \frac{Q}{A} = \frac{48 \times 1000}{2400}$
 $= 20 \text{ N/mm}^2$ (positive)

Shear modulus, $G = 25 \text{ kN/mm}^2$

Shear strains, $\phi = \frac{\tau}{G} = \frac{20}{25000} = 0.8 \times 10^{-3} \text{ radian}$
 $= 0.0458^\circ$

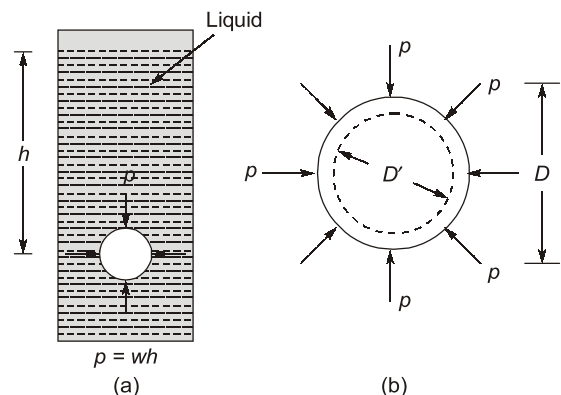
**Figure : 1.10****Practice Q.1.3**

A 40 mm cubical brass block is subjected to a shear force of 64 kN on one surface, other surface below this surface is fixed. If G for brass is 38 kN/mm², what are shear stress and shear strain developed in block?

Ans. [$\tau = 40 \text{ N/mm}^2$, $\phi = 0.00105 \text{ rad} = 0.060^\circ$]

1.5 Volumetric Stress and Volumetric Strain

Volumetric stress (or pressure, p) acts equally in all the directions (it acts over the volume) as shown in figure (a) pressure, $w \times h$ = Weight density of the liquid × depth of the immersed body from the free surface of the liquid as shown in figure 1.11 (a). Figure 1.11 (b) shows a spherical body of diameter D subjected to volumetric stress, p . Under the action of the pressure, p , the volume of the spherical ball is reduced, or the diameter D of the ball is reduced to D' , as shown in figure 1.11.

**Figure : 1.11**

$$\text{Diametral strain} = \frac{\text{Change in diameter}}{\text{Original diameter}} = \frac{-\delta D}{D} \quad (\text{reduction in diameter})$$

$$\text{Volume of the sphere,} \quad V = \frac{\pi}{6} \times D^3$$

$$\text{or, Partial derivative} \quad \delta V = \frac{\pi}{6} \times 3D^2 \delta D$$

$$\begin{aligned} \text{Volumetric strain,} \quad \frac{\delta V}{V} &= \left(\frac{\pi}{2} \times D^2 \right) \frac{\delta D}{V} \\ &= \left(\frac{\pi}{2} \times D^2 \right) \frac{\delta D}{V} \times 6 = \frac{3\delta D}{D} \end{aligned}$$

$$\text{Volumetric strain,} \quad \epsilon_V = \frac{3\delta D}{D} = 3 \times \text{diametrical strain}$$

$$\text{Now,} \quad p \propto \epsilon_V (\text{volumetric strain}) \quad (\text{as per Hooke's law})$$

Within the elastic limit, volumetric stress, p is linearly proportional to strain, ϵ_V .

$$\text{or} \quad \frac{p}{\epsilon_V} = K \quad (\text{Bulk modulus})$$

As per Pascal Law of Fluid Mechanics, **a liquid transmits pressure equally in all the directions.**

Question 1.4 A spherical ball of steel of diameter 150 mm goes down to a depth of 800 m in sea water. If the specific weight of sea water is 10.2 kN/m³ and the bulk modulus of steel is 175 kN/mm², determine change in diameter of the ball, change in its volume.

Solution: Specific weight of sea water,

$$w = 10.2 \text{ kN/m}^3$$

$$\text{Depth of sea water,} \quad h = 800 \text{ m}$$

$$\text{Pressure,} \quad p = wh = 10.2 \times 800 \text{ kN/m}^2$$

$$= 8160 \text{ kN/m}^2 = \frac{8160 \times 1000}{10^6} \text{ N/mm}^2 = 8.16 \text{ N/mm}^2$$

$$\text{Bulk modulus of steel,} \quad K = 175 \text{ kN/mm}^2$$

$$\text{Volumetric strain,} \quad \epsilon_V = \frac{p}{K} = \frac{8.16}{175 \times 10^3} = 0.0466 \times 10^{-3}$$

$$\begin{aligned} \text{Diametrical strain,} \quad \frac{\delta D}{D} &= \frac{1}{3} \times \epsilon_V \\ \epsilon_D &= \frac{0.0466 \times 10^{-3}}{3} = 0.0155 \times 10^{-3} \end{aligned}$$

$$\text{Diameter of ball,} \quad D = 150 \text{ mm}$$

$$\begin{aligned} \text{Change in diameter of the ball,} \quad \delta D &= \epsilon_D \times D = 0.0155 \times 10^{-3} \times 150 \\ &= 2.33 \times 10^{-3} \text{ mm} \end{aligned}$$

$$\text{Volume of the ball,} \quad V = \frac{\pi}{6} \times D^3 = \frac{\pi}{6} \times 150^3$$

Important Points to Remember

1. Whenever a body is subjected to a force at one end, there is equal and opposite reaction at the other end, i.e., a tensile force applied at one end will be resisted by a tensile reaction at the other end. Similarly, a compressive force applied at one end will be resisted by a compressive force reaction at the other end.
2. When a bar is subjected by external force at one end, then on internal section of the bar, there will be equal and opposite reaction. This internal reaction per unit area is stress.
3. A force acting perpendicular to a plane, is a direct normal force. If force is directed away from the plane, it is a tensile force, creating tensile stress in the body. Similarly, a force acting perpendicular to a plane, directed towards the plane is known as direct compressive force creating compressive stress in the body.
4. A force acting parallel (tangential) to the plane is a shear force. On the opposite plane, there will be an equal and opposite reaction. The shear force which tends to rotate the body in the clockwise direction is a positive shear force, causing positive shear stress (shear force per unit area). Similarly, if the tangential force in the plane tends to rotate the body in the anti-clockwise direction, it is a negative shear force, causing negative shear stress on the plane.
5. When a bar is subjected to axial stress σ , there will be change in the length of the bar. Change in length per unit length is known as longitudinal (or axial) strain, ϵ . If the stress, σ applied is within the elastic limits of the material, $\sigma \propto \epsilon$ (stress is proportional to strain). This is known as Hooke's law.

Similarly,

Shear stress, $\tau \propto \phi$ (shear strain)

Volumetric stress, $p \propto \epsilon_v$ (volumetric strain)

6. Say the stress in the bar is $+\sigma$ (tensile) stress, there will be extension in the length of the bar, \propto axial strain $+\epsilon$ (increase in length) there will be decrease in lateral dimension (say diameter in a round bar). There will be negative lateral strain.
Lateral strain, $\epsilon_l = -v\epsilon$, where v is Poisson's ratio of the material. Similarly, if there is negative axial strain (decrease in length), there will be increase in lateral dimension (breadth and thickness in a bar of rectangular section).
If ϵ is negative, lateral stress, ϵ_l will be positive.
7. Whenever a shear stress acts one plane, there will be automatic shear stress of opposite nature on the perpendicular plane, to maintain equilibrium of the body, i.e., +ve shear stress applied on one plane will be balanced by -ve shear stress on the perpendicular plane. This is known as complementary shear stress.
8. A load gradually applied on a body (starting from zero then going to the maximum), gradual stress is developed in the body and gradual load, W , area of cross-section is A , gradual stress, $\sigma = \frac{W}{A}$.
9. When whole of the load, W is applied all of a sudden on the body, stress produced due to sudden load is

$$\sigma_{\text{sudden}} = \frac{2W}{A}$$

10. When the load is applied with some velocity, the kinetic energy of the load is absorbed as strain energy in the body. Sometimes a large stresses are developed due to the impact load and the body may break.

11. Some extension in a rope or wire may occur due to the self weight of the rope, but it is very small.

$$\delta l_w = \text{due to self weight} = \frac{wL^2}{2E}$$

where w , specific weight density of the material of the bar. L , length and E , Young's modulus of the material.

12. In a uniformly tapered round bar, taking from diameter D at one end to diameter d at other end over axial length L .

Change in length, $\delta L = \frac{4PL}{\pi Ed}$, where P = axial force, E = Young's modulus



Objective Type Questions

- Q.1** What is the Poisson's ratio of aluminium?
 (a) 0.25 (b) 0.30
 (c) 0.33 (d) 0.35
- Q.2** On a plane, there are normal stress, σ and shear stress, τ if the resultant stress is inclined at an angle of 30° to the plane and σ is 50 N/mm^2 , what is the shear stress τ on the plane?
 (a) 43.3 N/mm^2 (b) 50 N/mm^2
 (c) 86.6 N/mm^2 (d) None of these
- Q.3** A circular bar of axial length 500 mm is subjected to axial stress s so that it extends by 0.4 mm . If $E = 200 \text{ GPa}$, what is σ ?
 (a) 160 MPa (b) 80 MPa
 (c) 40 MPa (d) None of these
- Q.4** A bar is subjected to axial strain ϵ , there is change in volume of the bar. The volumetric strain in the bar is 0.3ϵ , what is the Poisson's ratio of the material of the bar?
 (a) 0.5 (b) 0.4
 (c) 0.35 (d) 0.3
- Q.5** A bar of square section $a \times a$ is subjected to axial load P . On a plane inclined at 45° to the axis of the bar, shear stress on inclined plane will be
 (a) $\frac{2P}{a^2}$ (b) $\frac{P}{a^2}$
 (c) $\frac{P}{2a^2}$ (d) $\frac{P}{4a^2}$
- Q.6** Two tie rods are connected through a pin of a cross-sectional area 50 mm^2 . Pin is in double shear. If the tie rods carry a tensile load of 10 kN , what is the shear stress in the pin?
 (a) 50 MPa (b) 100 MPa
 (c) 125 MPa (d) None of these
- Q.7** A 20 m long wire rope is suspended vertically from a pulley. The wire rope weighs 15 N/m in length. Cross-sectional area of wire rope is 40 mm^2 . What is the maximum stress developed in wire?
 (a) 15 N/mm^2 (b) 10 N/mm^2
 (c) 7.5 N/mm^2 (d) 3.75 N/mm^2
- Q.8** A spherical ball of volume 1000 cm^3 , is subjected to a hydrostatic pressure of 100 N/mm^2 , bulk modulus of material of the ball is 200 kN/mm^2 . What is the change in volume of the ball?
 (a) 1.0 cc (b) 0.8 cc
 (c) 0.5 cc (d) 0.4 cc
- Q.9** A bar is subjected to axial stress $\sigma = 100 \text{ MPa}$. If E for the material is 100 GPa , what is the strain energy stored per unit volume.
 (a) 0.1 Nmm/mm^3 (b) 0.05 Nmm/mm^3
 (c) 0.04 Nmm/mm^3 (d) None of these
- Q.10** A weight of 50 N falls on a bar through height h . If the instantaneous stress developed in bar is 100 MPa and bar is of steel with $E = 200 \text{ GPa}$, what is the value of h ?

Length of the bar = 500 mm, Area of cross-section = 100 mm². (Neglect affect of change in length)

- (a) 25 mm (b) 50 mm
(c) 100 mm (d) None of these

Answers

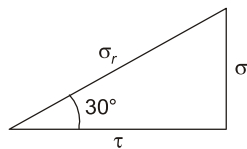
1. (c) 2. (c) 3. (a) 4. (c) 5. (c)
6. (b) 7. (c) 8. (c) 9. (b) 10. (a)

Explanations

1. (c)

Poisson's ratio of aluminium is 0.33.

2. (c)



$$\sigma = 30 \text{ N/mm}^2$$

$$\frac{\sigma}{\tau} = \tan 30^\circ$$

$$\begin{aligned}\tau &= \frac{\sigma}{\tan 30^\circ} \\ &= \frac{50}{0.577} = 86.6 \text{ N/mm}^2\end{aligned}$$

3. (a)

$$\begin{aligned}\sigma &= \frac{\delta L}{L} \times E \\ &= \frac{0.4}{500} \times 200 \times 1000 \\ &= 160 \text{ MPa}\end{aligned}$$

4. (c)

$$\begin{aligned}\epsilon_v &= \epsilon(1 - 2\nu) = 0.3\nu \\ \nu &= \frac{1 - 0.3}{2} = 0.35\end{aligned}$$

5. (c)

$$\begin{aligned}\text{Area of inclined plan} &= \frac{a^2}{0.707} \\ \sigma &= \frac{P}{a^2}\end{aligned}$$

$$\tau = \sigma \cos 45^\circ$$

$$\begin{aligned}\tau &= \frac{P}{a^2} \times 0.707 \times 0.707 \\ &= \frac{0.5P}{a^2}\end{aligned}$$

6. (b)

Pin in double shear

$$\begin{aligned}10 \times 1000 &= 2 \times 50 \times \tau \\ \tau &= 100 \text{ MPa}\end{aligned}$$

7. (c)

Length of wire rope = 20 m

$$\text{Total weight} = 20 \times 15 = 300 \text{ N}$$

$$\text{Area} = 40 \text{ mm}^2$$

$$\sigma = \frac{300 \text{ N}}{40} = 7.5 \text{ N/mm}^2$$

8. (c)

$$K = 200 \text{ kN/mm}^2,$$

$$\rho = 100 \text{ N/mm}^2$$

$$\begin{aligned}\frac{\rho}{K} &= e_v = \frac{100}{200 \times 10^3} \\ &= 0.5 \times 10^{-3}\end{aligned}$$

$$\begin{aligned}V &= 1000 \text{ cc}, \partial V = e_v V \\ &= 0.5 \times 10^{-3} \times 1000 = 0.5 \text{ cc}\end{aligned}$$

9. (b)

H = Strain energy per unit volume

$$\begin{aligned}&= \frac{\sigma^2}{2\epsilon} = \frac{100^2}{2 \times 100 \times 10^3} \\ &= 0.05 \text{ Nmm/mm}^3\end{aligned}$$

10. (a)

$$\begin{aligned}50 \times h &= \frac{100^2 \times 500 \times 100}{2 \times 200 \times 10^3} \\ &= 1250 \text{ Nmm} \\ h &= 25 \text{ mm}\end{aligned}$$

